COMPARISON OF ANALYTICAL CALCULATIONS AND GEANT4 SIMULATIONS OF THE GEOMETRICAL FACTOR AND MEAN PARTICLE PATH LENGTH IN THE LIULIN-MO DETECTOR ONBOARD THE TRACE GAS ORBITER

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Abstract: *The geometrical factor and mean particle path length are important parameters for the quantitative interpretation of particle-telescope measurements. In this work, analytical calculations of the mean particle path length in the Liulin-MO detector telescope are compared with numerical results obtained using Geant4 simulations. For a single 1 × 2 cm², 300 μm thick silicon detector in an isotropic particle field, Cauchy's theorem gives a mean path length of 574.16 μm, while for particles crossing both detectors of the telescope the analytical calculation gives approximately 320 μm. For the simplified four-PIN-Photodiod detector geometry, Geant4 gives mean path lengths of 571.42 and 587.06 μm for the individual detectors and 318.51 and 316.68 μm for particles crossing both detectors. The geometrical factor is determined numerically using a spherical source with a radius of 150 cm and cosine angular sampling. The effects of a 70 keV deposited-energy threshold and of the complete Liulin-MO + FREND instrument + Trace Gas Orbiter (TGO) satellite geometry are also investigated. The numerical results are in good agreement with the analytical estimates.*

СРАВНЕНИЕ НА АНАЛИТИЧНИ ИЗЧИСЛЕНИЯ И СИМУЛАЦИИ С GEANT4 НА ГЕОМЕТРИЧНИЯ ФАКТОР И СРЕДНИЯ ПЪТ НА ЧАСТИЦИТЕ В ДЕТЕКТОРА LIULIN-MO НА БОРДА НА TRACE GAS ORBITER

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Ключови думи: *Liulin-MO; геометричен фактор; среден изминат път; Geant4*

Резюме: *Геометричният фактор и средният път на частиците са важни параметри за количествената интерпретация на измерванията с телескопи от детектори. В настоящата работа аналитичните изчисления на средния път на частиците в детекторния телескоп на Liulin-MO са сравнени с числени резултати, получени чрез симулации с Geant4. За единичен силициев детектор с размери 1 × 2 cm² и дебелина 300 μm в изотропно поле теоремата на Коши дава среден път 574.16 μm, а за частици, преминаващи през двата детектора на телескопа, аналитичното изчисление дава приблизително 320 μm. За опростената геометрия с четири PIN детектора Geant4 дава 571.42 и 587.06 μm за отделните детектори и 318.51 и 316.68 μm за частиците, преминаващи през двата детектора. Геометричният фактор е определен числено чрез сферичен източник с радиус 150 cm и косинусово ъглово разпределение. Изследвани са също така ефектите от прага на отложена енергия от 70 keV, както и от пълната геометрия на инструмента Liulin-MO + FREND + спътника Trace Gas Orbiter (TGO). Получено е добро съгласие между числените резултати и аналитичните оценки.*

Introduction

The geometrical factor and mean particle path length are important characteristics of particle detectors. The geometrical factor relates the measured counting rate to the incident particle flux, while the particle path length in the sensitive volume is related to the deposited energy and is therefore important for the determination and interpretation of dosimetric quantities.

Liulin-MO is a separate dosimetric module of the Fine Resolution Epithermal Neutron Detector (FREND) onboard the ExoMars Trace Gas Orbiter (TGO) [7, 8]. Its silicon detector system measures the charged-particle component of the radiation environment and complements the neutron measurements performed by FREND.

For an idealized detector geometry, the mean particle path length can be calculated analytically. In the real instrument, however, particle transport is influenced by the detector materials and by the surrounding Liulin-MO, FREND and TGO structures. Geant4 simulations allow these effects to be included and also provide a numerical determination of the geometrical factor.

The aim of this work is to compare analytical and Geant4 results for the mean primary-particle path length and to determine the geometrical factor numerically. Calculations are performed for a simplified four-PIN-detector geometry and for the complete Liulin-MO + FREND + TGO model. The influence of a 70 keV deposited-energy threshold is also investigated.

Analytical Calculation of the Mean Particle Path Length

The geometrical factor can be determined directly using the expression given by Thomas and Willis [9]. For each of the Liulin-MO telescopes, a value of 1.38 cm² sr is obtained.

For particles crossing both detectors of the telescope, the path length in the upper detector depends on the angle of incidence (Fig. 1).

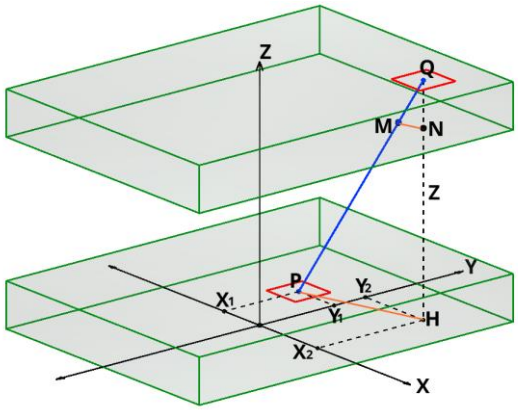


Fig. 1. Geometry used for the analytical calculation of the mean primary-particle path length through the Liulin-MO telescope

The mean path length can be obtained by weighting the individual particle paths by the corresponding contribution to the telescope counting rate. This gives

$$\bar{l} = \frac{QN}{QH} \frac{\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{QP\,dx_1 dy_1 dx_2 dy_2}{\left\{Z^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2\right\}^2}}{\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx_1 dy_1 dx_2 dy_2}{\left\{Z^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2\right\}^2}}, \quad (1)$$

where Z = QH = 2.14 cm is the geometrical distance used in the telescope model, while a = 1 cm, b = 2 cm and QN = 0.03 cm are the detector dimensions.

By expressing the length of QP in terms of the coordinates, we obtain

$$\bar{l} = \frac{QN}{QH} \frac{\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx_1 dy_1 dx_2 dy_2}{\left\{Z^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2\right\}^{3/2}}}{\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{dx_1 dy_1 dx_2 dy_2}{\left\{Z^2 + (x_1 - x_2)^2 + (y_1 - y_2)^2\right\}^{2}}}. \quad (2)$$

Numerical evaluation of Eq. (2) using Wolfram Mathematica gives

$$\bar{l} \approx 320\,\mu m. \quad (3)$$

In deriving Eq. (3), edge effects are neglected within the analytical approximation. For comparison, the mean path length for a single detector exposed to an isotropic particle field can be calculated using Cauchy's mean-chord theorem [6],

$$\bar{l}_{single} = 4\frac{V}{S}. \quad (4)$$

For a 1 × 2 × 0.03 cm³ silicon detector, V = 0.06 cm³ and S = 4.18 cm², giving

$$\bar{l}_{single} = 574.16\,\mu m. \quad (5)$$

Geant4 Numerical Model

The numerical calculations were performed using Geant4 [1–3]. Primary particles were generated uniformly over the surface of a sphere with radius R = 150 cm surrounding the detector geometry. Their directions were sampled according to a cosine angular distribution relative to the local inward normal, providing an isotropic particle field inside the sphere. The simulation geometry was constructed with CATIA Geometry Builder [4, 5].

A total of 10^8 **primary particles** were generated. The primary particles were **^{56}Fe ions with an energy of 1 GeV/nucleon**. The numerical geometrical factor was determined from the fraction of registered events relative to the total number of generated primary particles [10, 11]:

$$G = 4\pi^2 R^2 \frac{N_{registered}}{N_{generated}}. \quad (6)$$

The mean primary-particle path length was calculated by summing the Geant4 step lengths of each primary particle inside the corresponding silicon detector. Thus, the calculated quantity represents the actual transported primary-particle trajectory rather than only the straight geometrical chord.

Two geometrical configurations were investigated. The first consists of the four Liulin-MO PIN photodiodes and is used for direct comparison with the analytical model. The second includes the complete Liulin-MO geometry together with the surrounding FREND instrument and TGO spacecraft structures. The main components included in the Geant4 model are illustrated in (Fig. 2), while examples of simulated primary-particle trajectories are shown in (Fig. 3).

The calculations were performed both without an energy threshold and with a 70 keV deposited-energy threshold in the silicon detectors. This allows the influence of the experimental selection criterion and of the surrounding spacecraft geometry on the mean primary-particle path length to be evaluated.

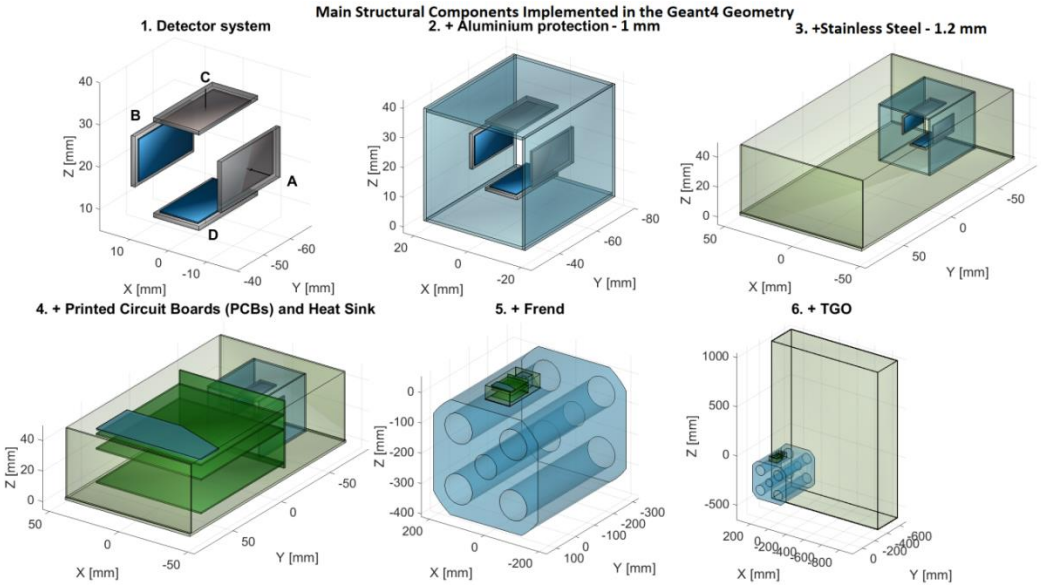


Fig. 2. Main structural components implemented in the Geant4 geometry: (1) Liulin-MO detector system; (2) aluminium protection; (3) iron protection; (4) printed circuit boards (PCBs) and heat sink; (5) FREND; and (6) TGO spacecraft

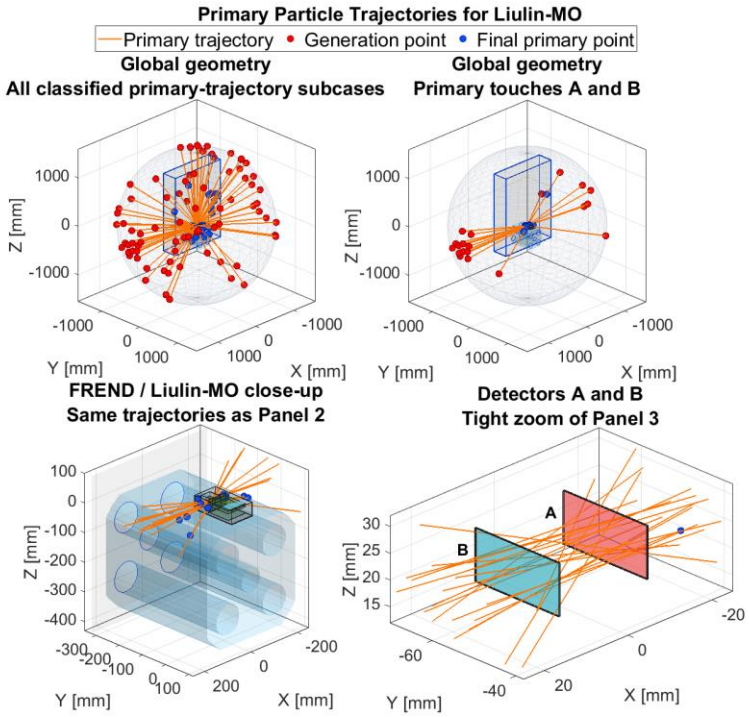


Fig. 3. Simulated primary-particle trajectories in the complete Liulin-MO + FREND + TGO geometry, from the global geometry to a detailed view of trajectories intersecting detectors A and B

Results and Discussion

For the simplified four PIN-Photodiod detector geometry without an energy threshold, the mean primary-particle path lengths are 571.42 ± 14.56 μm and 587.06 ± 19.85 μm for detectors A and B, respectively. These values are in good agreement with the analytical single-detector value of 574.16 μm. For particles crossing both telescope detectors, the corresponding mean path lengths decrease to 318.51 ± 2.99 and 316.68 ± 3.24 μm, in close agreement with the analytical telescope value of approximately 320 μm.

The reduction of the mean path length for particles crossing both detectors is a consequence of the telescope geometry. The coincidence condition restricts the range of accepted incidence angles and suppresses long paths produced by highly oblique trajectories. Consequently, the mean path length approaches the physical detector thickness of 300 μm.

Applying the 70 keV deposited-energy threshold changes the mean path lengths for the individual detectors to 629.52 and 663.68 μm, while the values for particles crossing both detectors remain practically unchanged at 318.51 and 316.68 μm. This indicates that the energy threshold modifies the broader single-detector population but has little influence on the geometrically restricted telescope population.

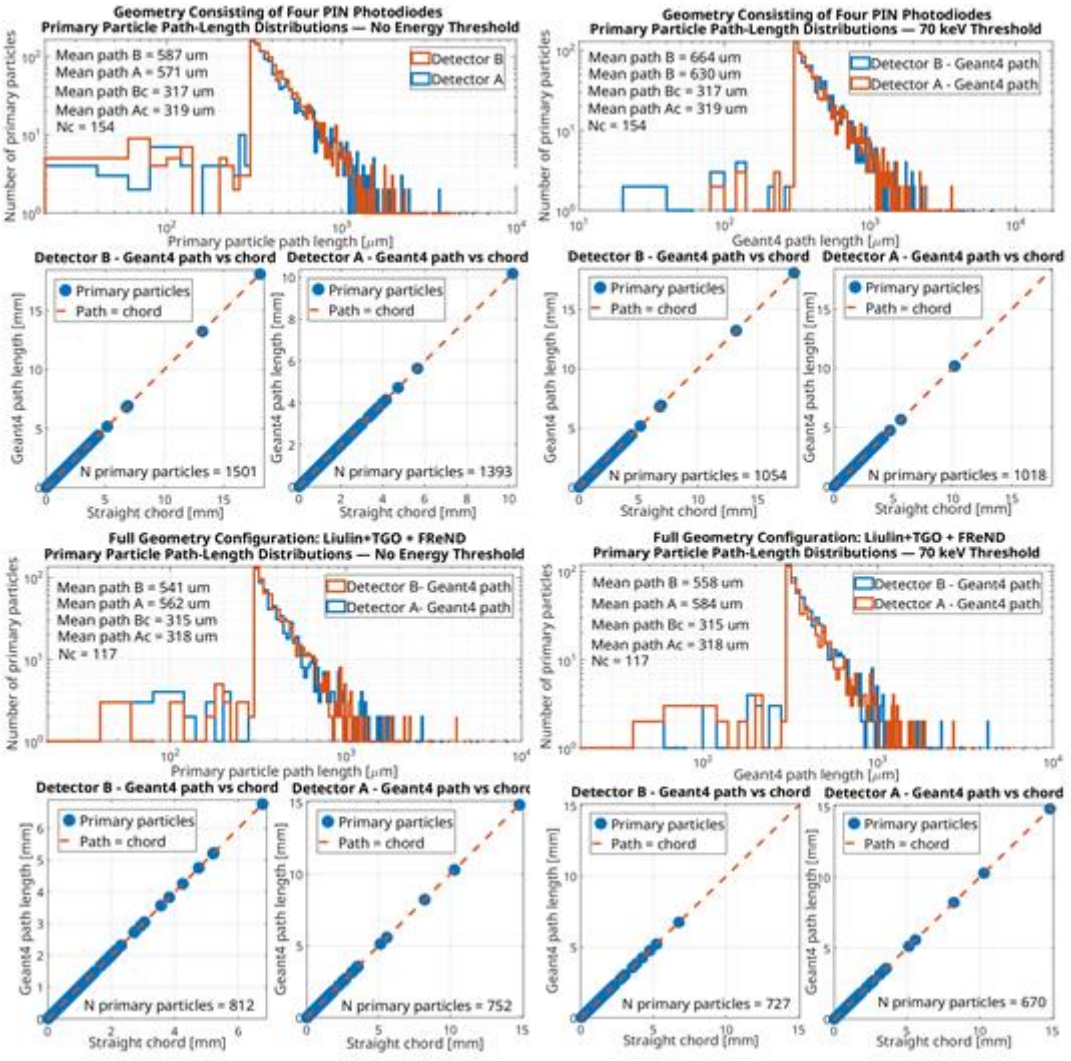


Fig. 4. Geant4 primary-particle path-length distributions and comparison between the Geant4 path length and the corresponding straight chord for the simplified four-PIN-photodiode geometry and the complete Liulin-MO + FREND + TGO geometry, without and with a 70 keV deposited-energy threshold. Ac and Bc denote the mean primary-particle path lengths in detectors A and B in coincidence mode, respectively, and Nc is the number of coincidence events.

For the complete Liulin-MO + FREND + TGO geometry, the mean path lengths without an energy threshold are 561.64 and 540.68 μm for detectors A and B. For particles crossing both detectors, the corresponding values are 317.71 and 315.01 μm. With the 70 keV threshold, the individual-detector values change to 584.39 and 558.39 μm, whereas the telescope values remain essentially unchanged. The surrounding structures therefore affect the broader population of particles reaching an individual detector, while the mean path length of particles satisfying the telescope condition remains close to the analytical value.

The comparison between the accumulated Geant4 primary-particle path and the corresponding straight geometrical chord, also shown in Fig. 4, demonstrates close agreement for most events. This confirms that the straight-trajectory approximation used in the analytical calculation is appropriate for the thin silicon detectors, while Geant4 additionally accounts for particle interactions, scattering and more complex trajectories.

For 10^8 generated primary particles and a spherical source radius of 150 cm, 154 initial trajectories intersect both telescope detectors. Using the Monte Carlo geometrical-factor expression gives $G_{MC} = 1.368\ cm^2$. The statistical uncertainty associated with the number of accepted trajectories is approximately 8.1%.

Conclusions

The analytical and Geant4 calculations show good agreement for the mean primary-particle path length in the Liulin-MO telescope. For a single silicon detector in an isotropic field, the analytical value of 574.16 μm is consistent with the numerical results. Requiring particles to cross both telescope detectors restricts the accepted angular range and reduces the mean path length to approximately 320 μm, reproduced by Geant4 within about 1%.

The 70 keV deposited-energy threshold and the surrounding Liulin-MO + FREND + TGO structures affect the mean path length of the broader single-detector particle population, but have only a small effect on particles satisfying the telescope condition. The results support the use of the analytical model and the Geant4 detector model for the interpretation of Liulin-MO measurements and for the determination of particle fluxes and dosimetric quantities.

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